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Name Key

Nth Roots and Rational Exponents

Evaluate the expression without using a calculator.

1) $\sqrt[4]{16} = \sqrt[4]{2^4} = \boxed{2}$

2) $(\sqrt[3]{64})^2$

$\sqrt[3]{64} = 4$ so $(\sqrt[3]{64})^2$

$= (4)^2$

$= \boxed{16}$

3) $9^{-\frac{5}{2}}$ ← negative exponent means it moves to denominator

$9^{-5/2} = \frac{1}{9^{5/2}} = \frac{1}{(\sqrt{9})^5} = \frac{1}{3^5} = \boxed{\frac{1}{243}}$

4) $216^{\frac{1}{3}} = \sqrt[3]{216} = \sqrt[3]{6^3} = \boxed{6}$

5) $\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = \boxed{-2}$

Properties of Rational Exponents

Simplify the expression. Assume all variables are positive.

1) $5^{\frac{1}{4}} \cdot 5^{-\frac{9}{4}}$

bases the same
add exponents

$5^{1/4} \cdot 5^{-9/4} = 5^{1/4 + -9/4} = 5^{-8/4} = 5^{-2} = \frac{1}{5^2} = \boxed{\frac{1}{25}}$

2) $(81x)^{\frac{1}{4}} = \sqrt[4]{81x} = \sqrt[4]{81} \cdot \sqrt[4]{x}$

$\sqrt[4]{81} = 3$ so $\boxed{3\sqrt[4]{x}}$

3) $\frac{\left(x^{\frac{1}{2}}y\right)^2}{x^{\frac{1}{2}}y^{\frac{3}{4}}}$

distribute

$\frac{x^{\frac{1}{2}\cdot 2}\,y^{2}}{x^{\frac{1}{2}}\,y^{3/4}} = \frac{x^{\frac{2}{2}}\,y^{2}}{x^{1/2}\,y^{3/4}}$

bases are the same
subtract exponents

$2 - 3/4 = \frac{8}{4} - 3/4 = 5/4$

$= x^{\frac{2}{2} - \frac{1}{2}}\,y^{2 - 3/4} = \boxed{x^{1/2}\,y^{5/4}}$

4) $\sqrt[6]{6x^6y^7z^{10}}$

$\sqrt[6]{6x^6y^6y^1z^6z^4}$

$\boxed{xyz\sqrt[6]{6y^1z^4}}$

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Power Functions and Function Operations

Let $f(x) = 2x - 4$ and $g(x) = x - 2$. Perform the indicated operation. State the domain.

1) $f(x) + g(x)$

$(2x+4) + (x-2)$

$2x+4+x-2$

$\boxed{3x-2}$ Domain = $\mathbb{R}$

2) $f(x) - g(x)$

$(2x-4)-(x-2)$

$2x-4-x+2$ Domain = $\mathbb{R}$

$\boxed{x-2}$

3) $g(f(x))$

outside inside

$(2x-4)-2 = 2x-4-2 = \boxed{2x-6}$ Domain = $\mathbb{R}$

Let $f(x) = 2x^{\frac{1}{2}}$ and $g(x) = x^4$. Perform the indicated operation. State the domain.

4) $\frac{f(x)}{g(x)}$

$\frac{2x^{1/2}}{x^4} = 2x^{1/2-4} = 2x^{-7/2} = \frac{2}{x^{7/2}} = \boxed{\frac{2}{(\sqrt{x})^7}}$

$\frac{1}{2}-4 = \frac{1}{2}-\frac{8}{2} = -\frac{7}{2}$

Domain = $\boxed{x>0}$

5) $g(x) \cdot f(x)$

$x^4 \cdot 2x^{1/2} = 2x^{4+1/2} = 2x^{9/2} = \boxed{2(\sqrt{x})^9}$

Domain = $\boxed{x \geq 0}$

6) $f(g(x))$

$2(x^4)^{1/2} = 2x^{4 \cdot 1/2} = 2x^2$

$\boxed{\text{Domain} = \mathbb{R}}$

Inverse Functions

Find the inverse function.

1) $f(x) = -2x + 1$

$y = -2x+1$

$x = -2y+1$

$x-1 = -2y$

$\boxed{y = \frac{x-1}{-2} \text{ or } y = -\frac{x}{2} + \frac{1}{2}}$

2) $f(x) = -x^4, x \geq 0$

$y = -x^4$

$x = -y^4$

$-x = y^4$

$\boxed{\sqrt[4]{-x} = y}$

3) $f(x) = 5x^3 + 7$

$y = 5x^3+7$

$x = 5y^3+7$

$x-7 = 5y^3$

$\frac{x-7}{5} = y^3$

$\boxed{\sqrt[3]{\frac{x-7}{5}} = y}$

4) Verify that $f(x) = -2x^5$ and $g(x) = \sqrt[5]{\frac{-x}{2}}$ are inverse functions.

$f(g(x))$ must $= x$

$-2\left(\sqrt[5]{\frac{-x}{2}}\right)^5$

$-2\left(\frac{-x}{2}\right)$

$\frac{-2 \cdot -x}{2} = \frac{2x}{2} = x$ ✓ inverses

Solving Radical Equations

Solve the equation. Check for extraneous solutions.

1) $\sqrt{x-4} = 6$

$(\sqrt{x-4})^2 = (6)^2$

$x - 4 = 36$

$\boxed{x = 40}$

2) $4x^{\frac{2}{3}} = 100$

$\frac{4x^{2/3}}{4} = \frac{100}{4}$

$x^{2/3} = 25$

$(\sqrt[3]{x^2})^3 = (25)^3$

$x^2 = 15625$

$\sqrt{x^2} = \sqrt{15,625}$

$\boxed{x = 125}$

3) $x^{\frac{5}{2}} - 10 = 22$

$x^{5/2} = 32$

$(\sqrt{x^5})^2 = (32)^2$

$\sqrt[5]{x^5} = \sqrt[5]{1024}$

$\boxed{x = 4}$

4) $\sqrt[3]{7x-9} + 11 = 14$

$(\sqrt[3]{7x-9})^3 = (3)^3$

$7x - 9 = 27$

$7x = 36$

$\boxed{\frac{36}{7} = x}$