Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date:\_\_\_\_\_\_\_\_\_\_\_ Block:\_\_\_\_\_

**Completing the square worksheet**

Finding the value of c needed to make an expression such as x2 + 6x + c into a perfect square trinomial is called **completing the square**.

To complete the square for the expression x2 + bx + c, replace c with $\frac{b}{2}$ . The perfect square trinomial is x2 + bx + $[\frac{b}{2}]^{2}$ and factors as (x + $\frac{b}{2})^{2}$.

**Example 1 - Complete the square to form a perfect square trinomial. Then factor the trinomial.**

|  |  |  |
| --- | --- | --- |
|  A  | **x2 + 12x + c** |   |
|   | Identify b. | b =                 |
|   | Find c. | c = $\frac{b}{2} $=                  |
|   | Write the trinomial. | x2 +        x +        |
|   | Factor the trinomial. | x2 +        x +        = (          )2 |
|  B  | **z2 − 26z + c** |   |
|   | Identify b. | b =                 |
|   | Find c. | c = $\frac{b}{2}$ =                  |
|   | Write the trinomial. | z2 +        z +        |
|   | Factor the trinomial. | z2 +        z +        = (           ) 2 |

**Example 2 - Solve the equation by completing the square.**

|  |  |  |
| --- | --- | --- |
|  A  | **x2 − 2x − 1 = 0** |   |
|   | Write the equation in the form x2 + bx = c. |                                                |
|   | Add $[\frac{b}{2}]^{2}$ to both sides of the equation. |                                                |
|   | Factor the perfect square trinomial. |                                                |
|   | Apply the definition of a square root. |                                                |
|   | Write two equations. |                                                |
|   | Solve the equations. |                                                |
|  |  |  |
|  B  | **x2 − 8x + 16 = 0** |   |
|   | Write the equation in the form x2 + bx = c |                                                |
|   | Add $[\frac{b}{2}]^{2}$ to both sides of the equation. |                                                |
|   | Factor the perfect square trinomial. |                                                |
|   | Apply the definition of a square root. |                                                |
|   | Write two equations. |                                                |
|   | Solve the equations. |                                                |

**PRACTICE**

**Complete the square to form a perfect square trinomial. Then factor the trinomial.**

|  |  |  |  |
| --- | --- | --- | --- |
| **1.** | m2 + 10m +        | **2.** | g2 − 20g +        |
|  |                                                  |  |                                                |
| **3.** | y2 + 2y +        | **4.** | w2 − 11w +        |
|  |                                                  |  |                                                |

**Solve the equation by completing the square.**

|  |  |  |  |
| --- | --- | --- | --- |
| **5.** | s2 + 15s = −56 | **6.** | r2 − 4r = 165 |
|  |                                                |  |                                                |
| **7.** | y2 + 19y + 78 = 0 | **8.** | x2 − 19x + 84 = 0 |
|  |                                                |  |                                                |
| **9.** | t2 + 2t − 224 = 0 | **10.** | x2 + 18x − 175 = 0 |
|  |                                                |  |                                                |
| **11.** | g2 + 3g = −6 | **12.** | p2 −3p = 18 |
|  |                                                |  |                                                |

## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date:\_\_\_\_\_\_\_\_\_\_\_ Block:\_\_\_\_\_

## Writing Quadratic Functions in Different Forms

* **A**  Write the function f(x) = 2(x − 4)2 + 3 in the form f(x) = ax2 + bx + c.

|  |
| --- |
| f(x) = 2(x − 4)2 + 3 |
| f(x) = 2(x2 −  + ) + 3 |   | Multiply to expand (x − 4)2. |
| f(x) = 2(x2) − (8x) + (16) + 3 |   | Distribute 2. |
| f(x) = 2x2 −  +  +3 |   | Multiply. |
| f(x) = 2x2 − 16x +  |   | Combine like terms. |

So, f(x) = 2(x − 4)2 + 3 is equivalent to .

* **B**  Write the function f(x) = x2 + 6x + 4 in vertex form.
Recall that the vertex form of a quadratic function is f(x) = a(x − h)2 + k. Write the given function in vertex form by completing the square.

|  |
| --- |
| f(x) = x2 + 6x + 4 |
|  |   | Set up for completing the square. |
| f(x) = (x2 + 6x +9) + 4 − 9 |   | Add a constant so the expression inside the parentheses is a perfect square trinomial. Subtract the constant to keep the equation balanced. |
| f(x) = (x + )2 + 4 − 9 |   | Write (x2 + 6x + 9) as a binomial squared. |
| f(x) = (x + 3)2 −  |   | Combine like terms. |

So, f(x) = x2 + 6x + 4 is equivalent to .

**Graph the function by first writing it in vertex form. Then give the maximum or minimum of the function and identify its zeros.**

 **A**  f(x) = x2 − 8x + 12

|  |  |  |  |
| --- | --- | --- | --- |
|  | **•** | Write the function in vertex form. |  |
|  |  |  | Set up for completing the square. |
|  |  | f(x) = (x2 − 8x + ) + 12 −  | Add a constant to complete the square. Subtract the constant to keep the equation balanced. |
|  |  | f(x) = (x − )2 + 12 − 16 | Write the expression in parentheses as a binomial squared. |
|  |  | f(x) = (x − 4)2 −  | Combine like terms. |
|  |  |  |  |
|  | **•** | The vertex is .Two points to the left of the vertex are(2, ) and (3, ) .Two points to the right of the vertex are(5, ) and (6, ).Describe the function’s properties. The minimum is. The zeros are and . |  |

 **B**  f(x) = −2x2 − 12x − 16

|  |  |  |  |
| --- | --- | --- | --- |
|  | **•** | Write the function in vertex form. |  |
|  |  | f(x) = (x2 + 6x) − 16 | Factor the variable terms so that the coefficient of x2 is 1. |
|  |  |  | Set up for completing the square. |
|  |  | f(x) = −2 (x2 + 6x + ) − 16 − (−2) | Complete the square. Since the constant is multiplied by −2, subtract the product of −2 and the constant to keep the equation balanced. |
|  |  | f(x) = −2 (x + )2 − 16 − (−2)9 | Write the expression in parentheses as a binomial squared. |
|  |  | f(x) = −2(x + 3)2 − 16 − () | Simplify (−2)9. |
|  |  | f(x) = −2(x + 3)2 +  | Combine like terms. |
|  |  |  |  |
|  | **•** | Sketch a graph of the function.The vertex is .Two points to the left of the vertex are ( −5, ) and (−4,).Two points to the right of the vertex are(−2, ) and (− 1, ).Describe the function’s properties.The maximum is .The zeros are and  . |  |

Completing the square to change a standard form equation into a vertex form is easy when a =1. But what happens when a ≠ 1? You can still complete the square but it gets more complicated. Instead, let’s do a trick.

When given f(x) = ax2+bx + c, start out by writing the vertex form.

* Write f(x) = a(x –h)2+k, substituting in the actual value for a from the original equation in standard form.
* Our h value is going to equal = $\frac{-b}{2a}$ (this comes from completing the square).
* Our k value is going to come from plugging h back into the original equation for x and simplifying.
* Plus in your new values for h and k.
* Viola! Vertex Form!

Let’s try this new way with our last example.

*f*(*x*) = −2*x*2 − 12*x* − 16

a = So, the vertex form is *f*(*x*) = a(x –h)2+k, which after substituting

h = $\frac{-b}{2a}$ = in the values to our left, we get *f*(*x*) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

k =

# ****PRACTICE****

**Graph each function by first writing it in vertex form. Then give the maximum or minimum of the function and identify its zeros.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **1.** | f(x) = x2 − 6x + 9 |        | **2.** | f(x) = x2 − 2x − 3 |
|  |  |  |  |  |
|    |  |      |    |  |
|  |  |  |  |  |
|  | **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** |  |    | **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** |
|  |  |  |  |  |
| **3.** | f(x) = −7 x2 − 14x |        | **4.** | f(x) = 3x2 − 12x + 9 |
|  |  |  |  |  |
|    |  |      |    |  |
|  |  |  |  |  |
|  | **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** |  |    | **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** |

* **5.** A company is marketing a new toy. The function s(p) = −50p2 + 3000p models how the total sales s of the toy, in dollars, depend on the price p of the toy, in dollars.
* **a.** Write the function in vertex form.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* **b.** What is the vertex of the graph of the function? What does the vertex represent in this situation?

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* **6.** A circus performer throws a ball from a height of 32 feet. The model h(t) = −16t2 + 16t + 32 gives the height of the ball in feet t seconds after it is thrown.
* **a.** Write the function in vertex form.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* **b.** What is the maximum height that the ball reaches?

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* **c.** What is a reasonable domain of the function? Explain.

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\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* **d.** What is the y-intercept of the function’s graph? What does it represent in this situation? What do you notice about the y-intercept and the value of c when the function is written in standard form?

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