**Lesson 1.4: Approximation and Error**

When we take measurements, we are usually reading some sort of scale. The scale of a ruler may have millimeters marked on it, but when we measure the length of an object, it is likely to fall between two divisions. So, we the reading, possibly to the nearest millimeter. Realistically, the ruler is only accurate to the nearest half a millimeter.

In other words, a measurement is only accurate to  of the smallest division on the scale.

 e.g. State the accuracy of the following measuring devices:

 (a) a set of scales marked in kilograms (b) a clock that has a “seconds” hand

 e.g. Rod used a tape measure with graduations in cm to measure his height. What is Rod’s height range if he estimated his height to be 188 cm?

 e.g. A rectangular block of wood was measured at 78 centimeters by 24 centimeters. What are the boundary limits of its perimeter?

 e.g. A cylinder has a radius of 5 cm and a height of 15 cm. Find the boundary values for the cylinder’s volume ().

An is a value given to a number which is close to, but not equal to, its true value.

 e.g. 36.428 97 is approximately 36.4.

An of a quantity is an approximation which has been found by judgment instead of carrying out a more accurate determination.

 e.g. 38.7 x 5.1 is estimated to be 40 x 5 = 200, whereas its true value is 197.37 and a good approximation using this true value is 197.

The in rounding is found by taking the difference between the estimated value and the true value.

 Error = estimated value – true value

The IB formula booklet gives the error as , where  represents the approximate value and  represents the exact value.

 e.g. Find the error if a retail sales figure of $ 458 110 is rounded to $ 460 000.

The is found by comparing the error with the actual value and expressing this as a percentage.

 Percentage error = 

The IB formula booklet gives the percentage error as , where  represents the approximate value and  represents the exact value.

We often find the percentage error, such that we find the size of the error and ignore its sign. The absolute value symbol keeps only the positive value of its contents. For example, .

 Absolute percentage error = 

 e.g. Find the absolute percentage error if a profit of $ 1 367 540 is rounded to $ 1.37 million.

Sometimes it is better to estimate rather than measure, providing that our estimation is reasonably accurate. The factors which influence our decision to estimate or measure might be: how much time it would take us to measure accurately and calculate; the difficulty of taking the measurements; and/or the degree of accuracy needed.