New test - October 04, 2015 *[37 marks]*

A particle is moving with a constant velocity along line L . Its initial position is A$(6 , -2 , 10)$. After one second the particle has moved to B$(9, -6 , 15)$.

1a. (i) Find the velocity vector, $\overrightarrow{\mathrm{AB}}$. *[4 marks]*

(ii) Find the speed of the particle.

Markscheme

(i) evidence of approach ***(M1)***

e.g. $\overrightarrow{\mathrm{AO}}+\overrightarrow{\mathrm{OB}}$, $\mathrm{B}-\mathrm{A}$, $\begin{pmatrix} 9-6 \\ -6+2 \\ 15-10 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$ (accept $(3,-4,5)$) ***A1 N2***

(ii) evidence of finding the magnitude of the velocity vector ***M1***

e.g. speed $=\sqrt{3^2+4^2+5^2}$

speed $=\sqrt{50}\ \left(=5\sqrt{2}\right)$ ***A1 N1***

[4 marks]

Examiners report

This question was quite well done. Marks were lost when candidates found the vector $\overrightarrow{\mathrm{BA}}$ instead of $\overrightarrow{\mathrm{AB}}$ in part (a) and for not writing their vector equation as an equation.

1b. Write down an equation of the line L . *[2 marks]*

Markscheme

correct **equation** (accept Cartesian and parametric forms) ***A2 N2***

e.g. $r=\begin{pmatrix} 6 \\ -2 \\ 10 \end{pmatrix}+t\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$, $r=\begin{pmatrix} 9 \\ -6 \\ 15 \end{pmatrix}+t\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$

[2 marks]

Examiners report

In part (b), a few candidates switched the position and velocity vectors or used the vectors $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{OB}}$ to incorrectly write the vector equation.

In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

2a. (i) Write down the coordinates of A. *[4 marks]*

(ii) Find the speed of the airplane in ms^{-1}.

Markscheme

(i) $(3, -4, 0)$ ***A1 N1***

(ii) choosing velocity vector $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ ***(M1)***

finding magnitude of velocity vector ***(A1)***

e.g. $\sqrt{(-2)^2 + 3^2 + 1^2}$, $\sqrt{4+9+1}$

speed $= 3.74$ $(\sqrt{14})$ ***A1 N2***

[4 marks]

Examiners report

Many candidates demonstrated a good understanding of the vector equation of a line and its application to a kinematics problem by correctly answering the first two parts of this question.

2b. After seven seconds the airplane passes through a point B. *[5 marks]*

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds.

Markscheme

(i) substituting $p = 7$ ***(M1)***

$\mathrm{B} = (-11, 17, 7)$ ***A1 N2***

(ii) **METHOD 1**

appropriate method to find $\overrightarrow{\mathrm{AB}}$ or $\overrightarrow{\mathrm{BA}}$ ***(M1)***

e.g. $\overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OB}}$, $\mathrm{A} - \mathrm{B}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -14 \\ 21 \\ 7 \end{pmatrix}$ or $\overrightarrow{\mathrm{BA}} = \begin{pmatrix} 14 \\ -21 \\ -7 \end{pmatrix}$ ***(A1)***

distance $= 26.2 \left(7\sqrt{14}\right)$ ***A1 N3***

METHOD 2

evidence of applying distance is speed × time ***(M2)***

e.g. 3.74×7

distance $= 26.2 \left(7\sqrt{14}\right)$ ***A1 N3***

METHOD 3

attempt to find AB^2 , AB ***(M1)***

e.g. $(3 - (-11))^2 + (-4 - 17)^2 + (0 - 7))^2$, $\sqrt{(3 - (-11))^2 + (-4 - 17)^2 + (0 - 7))^2}$

$\mathrm{AB}^2 = 686$, $\mathrm{AB} = \sqrt{686}$ ***(A1)***

distance $\mathrm{AB} = 26.2 \left(7\sqrt{14}\right)$ ***A1 N3***

[5 marks]

Examiners report

Many candidates demonstrated a good understanding of the vector equation of a line and its application to a kinematics problem by correctly answering the first two parts of this question.

Some knew that speed and distance were magnitudes of vectors but chose the wrong vectors to calculate magnitudes.

2c. Airplane 2 passes through a point C. Its position q seconds after it passes through C is given by *[7 marks]*

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}, a \in \mathbb{R}$.

The angle between the flight paths of Airplane 1 and Airplane 2 is $40°$. Find the two values of a.

Markscheme

correct direction vectors $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}$ ***(A1)(A1)***

$\left|\begin{matrix} -1 \\ 2 \\ a \end{matrix}\right| = \sqrt{a^2 + 5}$, $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix} = a + 8$ ***(A1)(A1)***

substituting ***M1***

e.g. $\cos 40° = \frac{a+8}{\sqrt{14}\sqrt{a^2+5}}$

$a = 3.21$, $a = -0.990$ ***A1A1 N3***

[7 marks]

Examiners report

Very few candidates were able to get the two correct answers in (c) even if they set up the equation correctly. Much contorted algebra was seen and little evidence of using the GDC to solve the equation. Many made simple algebraic errors by combining unlike terms in working with the scalar product (often writing $8a$ rather than $8+a$) or the magnitude (often writing $5a^2$ rather than $5+a^2$).

Distances in this question are in metres.

Ryan and Jack have model airplanes, which take off from level ground. Jack's airplane takes off after Ryan's.

The position of Ryan's airplane t seconds after it takes off is given by $r=\begin{pmatrix}5\\6\\0\end{pmatrix}+t\begin{pmatrix}-4\\2\\4\end{pmatrix}$.

3a. Find the speed of Ryan's airplane. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg magnitude of direction vector

correct working ***(A1)***

eg $\sqrt{(-4)^2+2^2+4^2}$, $\sqrt{-4^2+2^2+4^2}$

$6\ (\mathrm{ms}^{-1})$ ***A1 N2***

[3 marks]

Examiners report

[N/A]

3b. Find the height of Ryan's airplane after two seconds. *[2 marks]*

Markscheme

substituting 2 for t ***(A1)***

eg $0+2(4)$, $\boldsymbol{r}=\begin{pmatrix}5\\6\\0\end{pmatrix}+2\begin{pmatrix}-4\\2\\4\end{pmatrix}$, $\begin{pmatrix}-3\\10\\8\end{pmatrix}$, $y=10$

8 (metres) ***A1 N2***

[2 marks]

Examiners report

[N/A]

3c. The position of Jack's airplane s seconds after **it** takes off is given by $\boldsymbol{r}=\begin{pmatrix}-39\\44\\0\end{pmatrix}+s\begin{pmatrix}4\\-6\\7\end{pmatrix}$. *[5 marks]*

Show that the paths of the airplanes are perpendicular.

Markscheme

METHOD 1

choosing correct direction vectors $\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$ ***(A1)(A1)***

evidence of scalar product ***M1***

eg $\boldsymbol{a} \cdot \boldsymbol{b}$

correct substitution into scalar product ***(A1)***

eg $(-4 \times 4) + (2 \times -6) + (4 \times 7)$

evidence of correct calculation of the scalar product as 0 ***A1***

eg $-16 - 12 + 28 = 0$

directions are perpendicular ***AG N0***

METHOD 2

choosing correct direction vectors $\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$ ***(A1)(A1)***

attempt to find angle between vectors ***M1***

correct substitution into numerator ***A1***

eg $\cos\theta = \frac{-16-12+28}{|a||b|}$, $\cos\theta = 0$

$\theta = 90°$ ***A1***

directions are perpendicular ***AG N0***

[5 marks]

Examiners report

[N/A]

3d. The two airplanes collide at the point $(-23, 20, 28)$. *[5 marks]*

How long after Ryan's airplane takes off does Jack's airplane take off?

Markscheme

METHOD 1

one correct equation for Ryan's airplane ***(A1)***

eg $5 - 4t = -23,\ 6 + 2t = 20,\ 0 + 4t = 28$

$t = 7$ ***A1***

one correct equation for Jack's airplane ***(A1)***

eg $-39 + 4s = -23,\ 44 - 6s = 20,\ 0 + 7s = 28$

$s = 4$ ***A1***

3 (seconds later) ***A1 N2***

METHOD 2

valid approach ***(M1)***

eg $\begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$, one correct equation

two correct equations ***(A1)***

eg $5 - 4t = -39 + 4s,\ 6 + 2t = 44 - 6s,\ 4t = 7s$

$t = 7$ ***A1***

$s = 4$ ***A1***

3 (seconds later) ***A1 N2***

[5 marks]

Examiners report

[N/A]