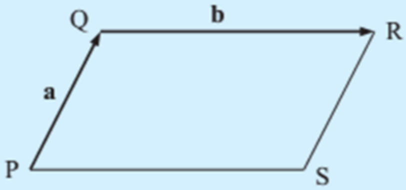
Example 2 — Self Tutor



PQRS is a parallelogram in which $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{QR} = \mathbf{b}$.

Find vector expressions for:

a $\overrightarrow{QP}$ **b** $\overrightarrow{RQ}$ **c** $\overrightarrow{SR}$ **d** $\overrightarrow{SP}$

a $\overrightarrow{QP} = -\mathbf{a}$ {the negative vector of $\overrightarrow{PQ}$}

b $\overrightarrow{RQ} = -\mathbf{b}$ {the negative vector of $\overrightarrow{QR}$}

c $\overrightarrow{SR} = \mathbf{a}$ {parallel to and the same length as $\overrightarrow{PQ}$}

d $\overrightarrow{SP} = -\mathbf{b}$ {parallel to and the same length as $\overrightarrow{RQ}$}

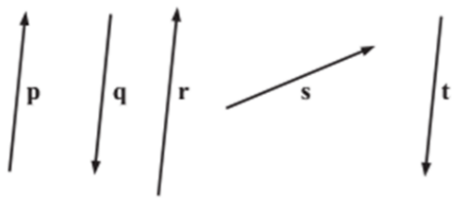
EXERCISE 12A.2

1 State the vectors which are:

a equal in magnitude **b** parallel

c in the same direction **d** equal

e negatives of one another.



3 Draw a scale diagram to represent the following vectors:

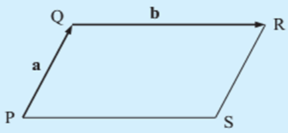
a a force of 30 Newtons in the NW direction

b a velocity of 36 m s^{-1} vertically downwards

c a displacement of 4 units at an angle of $15°$ to the positive x-axis

d an aeroplane taking off at an angle of $8°$ to the runway at a speed of 150 km h^{-1}.

Example 2 — Self Tutor



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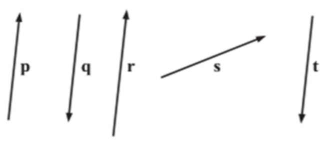
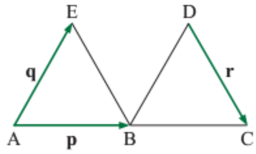


2 The figure alongside consists of two equilateral triangles. A, B, and C lie on a straight line.

$\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AE} = \mathbf{q}$, and $\overrightarrow{DC} = \mathbf{r}$.

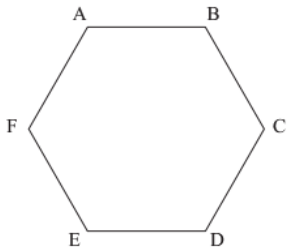
Which of the following statements are true?



a $\overrightarrow{EB} = \mathbf{r}$ **b** $|\mathbf{p}| = |\mathbf{q}|$ **c** $\overrightarrow{BC} = \mathbf{r}$

d $\overrightarrow{DB} = \mathbf{q}$ **e** $\overrightarrow{ED} = \mathbf{p}$ **f** $\mathbf{p} = \mathbf{q}$

3



ABCDEF is a regular hexagon.

a Write down the vector which:

i originates at B and terminates at C

ii is equal to $\overrightarrow{AB}$.

b Write down *all* vectors which:

i are the negative of $\overrightarrow{EF}$

ii have the same length as $\overrightarrow{ED}$.

c Write down a vector which is parallel to $\overrightarrow{AB}$ and twice its length.

DISCUSSION

- Could we have a zero vector?
- What would its length be?
- What would its direction be?

To construct $\mathbf{a}+\mathbf{b}$:

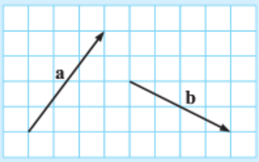
Step 1: Draw $\mathbf{a}$.

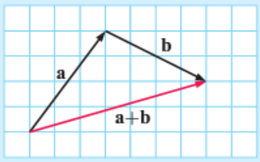
Step 2: At the arrowhead end of $\mathbf{a}$, draw $\mathbf{b}$.

Step 3: Join the beginning of $\mathbf{a}$ to the arrowhead end of $\mathbf{b}$. This is vector $\mathbf{a}+\mathbf{b}$.

Example 3 **Self Tutor**

Given $\mathbf{a}$ and $\mathbf{b}$ as shown, construct $\mathbf{a}+\mathbf{b}$.





THE ZERO VECTOR

Having defined vector addition, we are now able to state that:

The **zero vector** $\mathbf{0}$ is a vector of length 0.

For any vector $\mathbf{a}$: $\mathbf{a}+\mathbf{0}=\mathbf{0}+\mathbf{a}=\mathbf{a}$

$\mathbf{a}+(-\mathbf{a})=(-\mathbf{a})+\mathbf{a}=\mathbf{0}$.

When we write the zero vector by hand, we usually write $\overrightarrow{0}$.

Example 4 **Self Tutor**

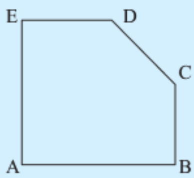
Find a single vector which is equal to:

a $\overrightarrow{BC}+\overrightarrow{CA}$

b $\overrightarrow{BA}+\overrightarrow{AE}+\overrightarrow{EC}$

c $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}$

d $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}$

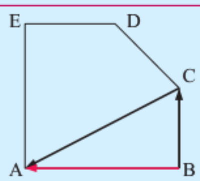


a $\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{BA}$ {as shown}

b $\overrightarrow{BA}+\overrightarrow{AE}+\overrightarrow{EC}=\overrightarrow{BC}$

c $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{AA}=\mathbf{0}$

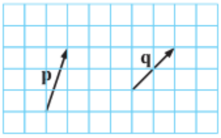
d $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}=\overrightarrow{AE}$



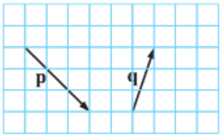
EXERCISE 12B.1

1 Use the given vectors **p** and **q** to construct $\mathbf{p}+\mathbf{q}$:

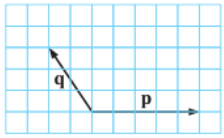
a



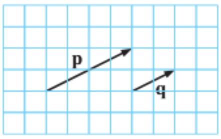
b



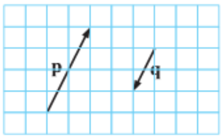
c



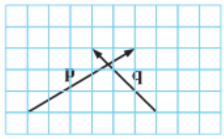
d



e



f

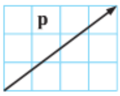


2 Find a single vector which is equal to:

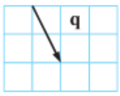
a $\overrightarrow{AB}+\overrightarrow{BC}$ **b** $\overrightarrow{BC}+\overrightarrow{CD}$ **c** $\overrightarrow{AB}+\overrightarrow{BA}$

d $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$ **e** $\overrightarrow{AC}+\overrightarrow{CB}+\overrightarrow{BD}$ **f** $\overrightarrow{BC}+\overrightarrow{CA}+\overrightarrow{AB}$

3 **a** Given



and

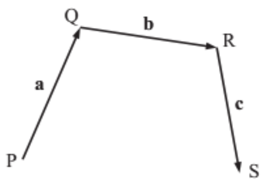


use vector diagrams to find:

i $\mathbf{p}+\mathbf{q}$ **ii** $\mathbf{q}+\mathbf{p}$.

b For any two vectors **p** and **q**, is $\mathbf{p}+\mathbf{q}=\mathbf{q}+\mathbf{p}$?

4 Consider:



One way of finding $\overrightarrow{PS}$ is:

$$\overrightarrow{PS}=\overrightarrow{PR}+\overrightarrow{RS}$$
$$=(\mathbf{a}+\mathbf{b})+\mathbf{c}.$$

Use the diagram to show that $(\mathbf{a}+\mathbf{b})+\mathbf{c}=\mathbf{a}+(\mathbf{b}+\mathbf{c})$.

5 An aeroplane in calm conditions is flying at 800 km h^{-1} due east. A cold wind suddenly blows from the south-west at 35 km h^{-1}, pushing the aeroplane slightly off course.

Things to think about:

a How can we illustrate the plane's movement and the wind using a scale diagram?

b What operation do we need to perform to find the effect of the wind on the aeroplane?

c Can you use a scale diagram to determine the resulting speed and direction of the aeroplane?